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A BAYESIAN FRAMEWORK FOR ESTIMATING WEIBULL DISTRIBUTION PARAMETERS: APPLICATIONS IN FINANCE, INSURANCE, AND NATURAL DISASTER ANALYSIS.

Mohammad Lawal Danrimi[1] and *Hamza Abubakar[2]
[1]Department of Accounting, Umaru Musa Yar'adua Univeristy P.M.B 2218, Katsina, Nigeria.
[2]Department of Mathematics, Isa Kaita College of Education P.M.B 5007, Katsina, Nigeria.

Corresponding Address: zeeham4u2c@yahoo.com

Abstract

This research presents a Bayesian framework for parameter estimation in the two-parameter Weibull distribution, with applications in finance and investment data analysis. The Weibull distribution is widely used for modeling stock pricing movements and making uncertain predictions in financial datasets. The proposed Bayesian approach assumes a gamma prior distribution for the scale parameter, with a known shape parameter. A simulation study using simulated financial data compares the Bayesian method with maximum likelihood estimators in terms of accuracy, error accumulation, and computational time across various sample sizes and parameter values. Results indicate the Bayesian approach performs similarly to maximum likelihood for small samples, while demonstrating computational efficiency for larger financial datasets. The proposed Bayesian model's application to simulated financial data showcases its practical relevance in real-world scenarios. This Bayesian framework offers a valuable tool for handling uncertainty and making informed decisions in financial data analysis, providing robust parameter estimation and uncertainty quantification in finance and investment domains.

Keywords: *Weibull distribution, Bayesian estimation, Maximum likelihood estimation, financial data analysis, Insurance claims, Root Mean Square Error, Mean Absolute Error.*

INTRODUCTION

Insurance companies play a pivotal role in modern society by offering financial protection against various risks (Hassani et al., 2020). Precise modeling and prediction of insurance claims data are indispensable for effective risk management, pricing policies, and ensuring the long-term sustainability of insurance companies. Bayesian modeling approaches have garnered considerable attention for their adaptability in incorporating prior knowledge and updating beliefs based on observed data (Abubakar & Sabri, 2023; Hamza & Sabri, 2022; van de Schoot et al., 2021). Among these approaches, the utilization of the Weibull distribution has displayed promising outcomes in modeling insurance claims data, as it can effectively capture the heavy-tailed and skewed characteristics often observed in real-world insurance data (Denuit et al., 2007; Ghosh et al., 2016).

In actuarial science, claims data is often characterized by its distributional properties, such as heavy tails, skewness, and heterogeneity, as highlighted by various studies (Bernardi et al., 2012; Miljkovic & Grün, 2016). Traditional parametric models like the Gaussian distribution often struggle to accurately capture these features, resulting in suboptimal predictions and risk assessments. To address these shortcomings, researchers and practitioners have turned to alternative distributions like the Weibull distribution, which finds extensive application in survival analysis and reliability modeling (Almalki & Yuan, 2013; Bala & Napiah, 2020; Jazi et al., 2010).

The utility of the Weibull distribution extends to various fields, including finance, insurance, and natural disasters. Particularly in applied sciences like survival and reliability engineering, data modeling plays a crucial role in accelerating predictions and extracting value from data. Modeling data reliability is essential for informed decision-making processes (Méndez-González et al. 2019; Upadhyay & Gupta 2010; Shakhatreh, Lemonte, & Moreno–Arenas 2019).

In the realm of statistical distributions, which describe the probabilistic behavior of random phenomena, researchers have devoted considerable attention to various distributions, such as the normal distribution, Pareto distribution, gamma distribution, Maxwell distributions, and Poisson distribution (Carrasco et al., 2008; Cordeiro et al., 2014; De Pascoa et al., 2011; Emmert-Streib & Dehmer, 2019; Jazi et al., 2010). These distributions have proven their usefulness in different domains, including science, engineering, business, and finance (Abubakar & Muhammad Sabri, 2021; Abubakar & Sabri, 2022, 2023; Jiang, 2020; Kobayashi et al., 2011; Yu, 2022).

The Bayesian approach presents a natural framework for integrating prior knowledge and updating beliefs based on observed data (Kim & Park, 2019). By specifying suitable prior distributions for the Weibull parameters, Bayesian modeling enables the estimation of posterior distributions, offering a comprehensive representation of the uncertainty associated with parameter estimates (Deng & Aminzadeh, 2022; van de Schoot et al., 2021). Moreover, Bayesian methods provide flexibility in model selection, comparison, and averaging, leading to robust inference and improved decision-making in insurance applications (Allenbrand & Sherwood, 2023; Lesmana et al., 2018; Wu et al., 2021).

The combination of Bayesian modeling and the Weibull distribution in insurance analysis shows great potential for enhancing risk assessment and decision-making processes. This study aims to contribute to the existing literature by evaluating the effectiveness of the Bayesian approach in modeling insurance claims data using the Weibull distribution, thereby providing valuable insights for the insurance industry (Abubakar & Sabri, 2023; Ching & Yip, 2022; Deng & Aminzadeh, 2022; Hamza & Sabri, 2022).

In the actuarial literature, there has been considerable interest in newly proposed models for insurance applications, particularly in survival modeling and analysis. The availability of computational resources has facilitated the utilization of statistical models in insurance analysis, attracting increased attention (Ahmad et al., 2020). This study emphasizes the significance of statistical models as a comprehensive tool for analyzing the claims process. Previous research has explored the application of Bayesian methods in estimating parameters of various statistical distributions. For instance, Kaminskiy and Krivtsov (2005) suggested the use of interval assessment for reliability expressions, which can be more accessible compared to direct estimation of Weibull parameters.

Actuarial science heavily relies on statistical models and distributions to analyze and predict risks and events in insurance and finance. Recent literature reviews have highlighted the use of various statistical distributions in actuarial data analysis. For instance, Antonio and Beirlant (2007) studied the use of generalized linear mixed models (GLMMs) for analyzing repeated measurements or longitudinal data in actuarial statistics. These models effectively handle data with fixed and random effects, and the article provided implementation details in SAS and WinBugs.

Another study by Adcock et al. (2015) focused on skewed distributions in finance and actuarial science, particularly the use of skew-normal and skew-Student distributions. These distributions were found to be interpretable, tractable, and applicable in risk measurement, capital allocation, portfolio selection, asset pricing, and actuarial research.

Ghitany et al. (2018) proposed a new generalization of the Pareto distribution to better fit data with long tails, particularly in actuarial statistics and finance. They applied this generalization to earthquake insurance data.

In Poudyal's study (2021), robust fitting of a single-parameter Pareto distribution was addressed, considering challenges like truncation, censoring, and other transformations in actuarial claim severity data. The study introduced the method of truncated moments (MTuM) as a robust estimation technique for this distribution, comparing its performance to maximum likelihood estimation.

Benatmane et al. (2021) explored the composite Rayleigh-Pareto (CRP) distribution as a potential fit for heavy-tailed insurance claims data. They compared its statistical properties to the composite lognormal-Pareto distribution using simulated examples and real fire insurance data sets.

Aljohani et al. (2021) introduced the uniform Poisson-Ailamujia distribution, an asymmetric discrete model suitable for various applications in public health, biology, sociology, medicine, and agriculture. The study investigated its statistical properties and discussed parameter estimation methods.

Riad et al. (2023) proposed the Kavya-Manoharan power Lomax (KMPLo) distribution as a new heavy-tailed model for financial data. They compared it to other models and evaluated its performance using simulations and insurance loss data. The KMPLo distribution demonstrated flexibility and affordability compared to competing models.

Several studies have also explored the Bayesian approach for parameter estimation in the Weibull distribution. For example, Sultan et al. (2014) investigated Bayes estimators for the parameters of the inverse Weibull distribution, using universal entropy and squared error loss functions. They evaluated the performance of the proposed estimators based on predicted risks.

Overall, these studies contribute valuable insights and tools for modeling and analyzing complex data structures with heavy tails in various fields, including actuarial science, finance, and public health.

Yanuar et al. (2019) conducted research on Bayesian inference for the scale parameters of the Weibull distribution and utilized different prior distributions, including Jeffreys' prior, inverse gamma, and non-informative priors. They explored theoretical aspects of posterior distributions based on these priors and applied them to generated data to compare the performance of Bayes estimators. Köksal et al. (2019) focused on the fitness of Bayes estimators for the scale and shape parameters of the Weibull distribution, assuming Gamma priors. They discovered that a continuous conjugate joint prior distribution for computing Bayes estimates does not exist in the Weibull distribution. Almetwally and Almongy (2021) adopted a Bayesian estimation approach with a square error loss function to estimate generalized power Weibull parameters. The study also considered optimal censoring strategies based on mean squared error, bias, and relative efficiency criteria. Tung et al. (2021) investigated Bayesian analysis and the effectiveness of Gibbs sampling, assessing convergence using metrics like Raftery-Lewis, Geweke, and Gelman-Rubin. The study aimed to evaluate the performance and convergence of the algorithm. Shakhatre (2021) focused on estimating the differential entropy of the Weibull distribution using multiple non-informative prior distributions, developing reference prior and probability matching prior approaches for the differential entropy estimation.

These studies collectively demonstrate the growing interest in using Bayesian methods for parameter estimation in the Weibull distribution. While frequentist estimation of Weibull distribution parameters has been extensively explored, limited research has been conducted on Bayesian inference with a gamma prior, especially in the context of claims modeling. Therefore, further exploration and investigation into Bayesian methods for Weibull distribution parameter estimation, specifically in claims modeling, are needed.

Furthermore, Bayesian approaches have been employed in insurance data analysis beyond the Weibull distribution. For example, Bernardi et al. (2012) used a Bayesian approach with a mixture of skew normal distributions to estimate loss distributions in insurance data. This research emphasized the importance of statistical models and advanced techniques like Bayesian approaches for analyzing insurance claims data and evaluating risk measures such as Value-at-Risk and Expected Shortfall probability. These studies contribute to the advancement of robust methodologies by highlighting the significance of Bayesian techniques and statistical models in reliability modeling and insurance data analysis, providing an alternative to traditional frequentist methods for handling complex data structures and risk assessment.

Despite the increasing interest in Bayesian modeling techniques combined with the Weibull distribution for analyzing insurance claims data, there is still a need for comprehensive simulation studies to thoroughly assess the performance of these approaches. This study aims to address this gap by conducting a rigorous simulation study that systematically investigates the effectiveness of Bayesian models in modeling insurance claims data using the Weibull distribution.

The primary objective of this study is to evaluate the performance of Bayesian models under various data characteristics, such as sample size, shape parameter, and scale parameter. By manipulating these factors, the study aims to assess the robustness and accuracy of Bayesian models in capturing the complexities inherent in insurance claims data. Additionally, the results obtained from the Bayesian approach will be compared with those derived from traditional frequentist approaches to provide a comprehensive analysis.

To achieve these objectives, the study will draw upon existing literature and established simulation techniques. Simulated data will be generated based on different scenarios that reflect a wide range of practical situations encountered in insurance claims analysis. Through an in-depth analysis of the simulated data, the performance of Bayesian models will be evaluated in terms of parameter estimation, model fit, and predictive accuracy.

This study builds upon the work of previous researchers who have made significant contributions to Bayesian modeling and statistical model assessment. By incorporating their findings and methodologies into the simulation study, the aim is to contribute to the existing body of knowledge and provide practical guidance for modeling insurance claims data using the Weibull distribution.

The objectives of this study include:

1. Constructing an insurance claims model based on the Weibull distribution, with the shape parameter following a Gamma distribution.
2. Applying a Bayesian model framework to estimate the parameters of the Weibull distribution model developed in objective 1.
3. Comparing the performance of the Bayesian estimator with the maximum likelihood estimator (MLE) based on simulated insurance claims data.

These objectives aim to investigate the effectiveness and performance of the Bayesian approach in modeling insurance claims data using the Weibull distribution. The study aims to provide insights and recommendations for practitioners in the insurance industry based on the comparison between the Bayesian and MLE approaches. The rest of this paper is structured as follows: In Section 2, we present the materials and methods, including the Weibull distribution model, likelihood function, Bayesian estimation method, prior assumptions on insurance claim amount data, and the method of estimating expected future insurance claims

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amounts from the posterior distribution. In Section 3, we present a simulation study on the Weibull distribution as claims amount, assuming a gamma prior for the scale parameter. In Section 4, we conduct a performance evaluation of the Bayesian approach and the maximum likelihood via simulation study. The results and discussion are presented in Section 5. Finally, this paper concludes in Section 6.

MATERIALS AND METHODS

In this study, the expected insurance claims in the financial data were estimated using the Bayesian approach. The likelihood function of the best fit model for the claims return amounts was incorporated with a gamma prior distribution to determine the posterior distribution. The expected insurance claims amount was then determined by the expectation of the posterior distribution. By incorporating the prior information from the gamma distribution and updating it with the likelihood function, a robust and informative estimation of the expected claims in the financial data was provided using the Bayesian approach. The experimental methodology aims to compare the Bayesian approach with traditional maximum likelihood methods for modeling financial data using the Weibull distribution. Let's define the key mathematical components of this statement:

1. Bayesian Approach for Modeling Financial Data: Let θ be the vector of parameters of the Weibull distribution ($\theta = [\alpha, \beta]$, where α is the shape parameter and β is the scale parameter).

The Bayesian framework involves updating our prior beliefs about θ using the observed financial data, D, to obtain the posterior distribution, $P(\theta|D)$.

Bayes' theorem is utilized to calculate the posterior distribution: $P(\theta|D) \propto P(D|\theta) * P(\theta)$,

where $P(D|\theta)$ is the likelihood function representing the probability of observing the financial data D given the parameter values θ, and $P(\theta)$ is the prior distribution representing our initial beliefs about the parameter values before observing the data.

2. Maximum Likelihood Method for Modeling Financial Data: The maximum likelihood estimation involves finding the parameter values that maximize the likelihood function $P(D|\theta)$ given the observed financial data D. The likelihood function for the Weibull distribution is defined as: $L(\theta|D) = \prod(f(x_i;\theta))$, where $f(x_i;\theta)$ is the probability density function of the Weibull distribution for the data point xi with parameter values θ.

3. Suitability for Modeling Financial Data: By comparing the Bayesian approach and maximum likelihood method, we can assess the suitability of the Bayesian approach for modeling financial data. The Bayesian approach allows us to incorporate prior knowledge or assumptions about the parameter values, which can be particularly valuable in the context of financial modeling where domain expertise may exist. It allows for uncertainty quantification and provides a posterior distribution, which provides a more comprehensive representation of parameter uncertainty compared to point estimates obtained from the maximum likelihood

method.

4. Potential Advantages over Traditional Maximum Likelihood Methods: The Bayesian approach can handle small sample sizes effectively and produce more stable parameter estimates, which is particularly relevant for financial data that may be limited in size. It provides a principled way to incorporate prior information, which can lead to more accurate parameter estimates, especially when data are sparse or noisy. The Bayesian approach offers a natural way to update parameter estimates as new data becomes available, making it suitable for adaptive modeling in dynamic financial environments. It allows for the integration of expert knowledge and subjective judgments, enabling better decision-making in finance and investment domains.

By conducting the experimental methodology and analyzing the results, we can gain valuable insights into how the Bayesian approach performs in modeling financial data and its potential advantages over traditional maximum likelihood methods, thereby informing better modeling practices in finance and investment analysis.

Simulation Study on Weibull Distribution with shape parameter following a gamma Distribution

In this section, we conduct a simulation study to evaluate the behavior of Bayesian estimators for a finite sample size (n) using the Weibull distribution. We compare the performance of Bayesian estimators with maximum likelihood estimators for the shape and scale parameters of the Weibull distribution in terms of error accumulation criteria. We assume that the shape parameters follow Gamma priors.

The simulation study is implemented using the Python programming language, and the code has been developed by the authors. For the simulation, we generate random claim amounts from the Weibull distribution using Python software. The simulation is performed following these steps:

1. Define Prior Distributions: We specify Gamma prior distributions for the shape parameters of the Weibull distribution. The Gamma distribution parameters are chosen based on domain knowledge or initial beliefs.

2. Generate Sample Data: We simulate a sample dataset of size n, representing claim amounts from the Weibull distribution using Python's random number generation capabilities. The sample data will be used for parameter estimation.

3. Bayesian Parameter Estimation: For Bayesian estimation, we calculate the posterior distribution of the parameters (shape and scale) based on the prior distributions and the observed data. The Bayesian approach allows us to update our prior beliefs using the sample data, resulting in more accurate parameter estimates.

4. Maximum Likelihood Parameter Estimation: For maximum likelihood estimation, we find

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the parameter values (shape and scale) that maximize the likelihood function given the observed data. The maximum likelihood method provides point estimates for the parameters.

5. Evaluation of Estimators: We compare the Bayesian and maximum likelihood estimators in terms of their performance metrics, such as mean squared error (MSE), bias, and variance. These metrics will provide insights into the accuracy and precision of the estimators.

6. Repeat Simulation: We repeat the simulation process multiple times to obtain a robust assessment of the estimators' performance. Each simulation run involves generating a new random sample from the Weibull distribution.

The simulation study allows us to assess the effectiveness of Bayesian estimators when the shape parameter follows a Gamma distribution. By comparing the Bayesian approach with maximum likelihood estimation, we can determine which method yields more reliable parameter estimates for the Weibull distribution. The findings from this simulation study will contribute to understanding the strengths and limitations of each approach and provide valuable insights for parameter estimation in practical applications involving financial and insurance data.

Step 1. Generate sample of sizes $n = 50, 80, 150, 200, \ldots 1000$ from the random claims distribution.

Step 2. Compute the MLE for the proposed model parameters.

Step 3. For each sample size, samples with $\beta = 1, \alpha = 1, 1.5, 2$ and 2.5 values estimate Weibull parameters

Step4. Compute the RMSEs, MAE and CPU time.

Performance evaluation metrics

The performance of the Bayesian approach has been evaluated and compared with the maximum likelihood methods in terms of error accumulation. A model with lower error accumulation is considered as the best fit model to the claims distribution data set. The following formula have been used for RMSE and MAE

$$RMSE = \sqrt{\sum_{t=1}^{n} \frac{\left(b_t - \hat{b}_t\right)^2 + \left(a_t - \hat{a}_t\right)^2}{n}}$$

$$MAE = \frac{\sum_{t=1}^{n} \left|b_t - \hat{b}_t\right| + \left|a_t - \hat{a}_t\right|}{n}$$

where β_t, β_t, α_t and α_t are the exact value of the shapes parameters, estimated values of the shape parameter, exact value of the scale parameter and the estimated values of scale parameter respectively.

RESULT AND DISCUSSION

Simulation results have been reported in Tables 1, Figures 1 until 4, displayed the RMSE of various estimation methods of Weibull distribution while Figure 6 until 8 displayed the MAE of various methods of Weibull distribution understudy for different values of shapes and scale parameters with different samples sizes n= 50, 100,150, 200, 500, 800.

Table 1. The Various Estimators of $\beta = 1, \alpha = 1, 1.5, 2,$ 2.5 and 3 parameters of Weibull distribution

β	α	N	Bayesian Method		Maximum Likelihood methods	
			β_t	α_t	β_t	α_t
1	1	50	1.0019	1.0535	1.0087	0.9235
		100	1.0035	1.0307	1.0034	1.0067
		150	1.0069	1.0102	1.0019	1.0029
		200	1.0101	1.0105	1.0021	1.0016
		300	1.0202	1.0307	1.0201	1.0036
		500	1.0411	1.0401	1.0111	1.0076
		800	1.0910	1.0675	1.0521	1.0106
		1000	1.0915	1.0835	1.0723	1.0110
1	1.5	50	1.0025	1.5351	1.0078	1.5015
		100	1.0014	1.5197	1.0054	1.5009
		150	1.0011	1.5103	1.0016	1.5005
		200	1.0012	1.5571	1.0020	1.5012
		300	1.0024	1.5001	1.01020	1.51015
		500	1.0032	1.5171	1.0410	1.5090
		800	1.0112	1.5072	1.0810	1.5005
		1000	1.0167	1.5032	1.0940	1.5015
1	2	50	1.0027	2.0651	1.0187	2.0105
		100	1.0012	2.0117	1.0033	2.0046
		150	1.0005	2.0123	1.0013	2.0053
		200	1.0011	2.0005	1.0008	2.0007
		300	1.0091	2.0054	1.0027	2.0064
		500	1.0101	2.0096	1.0087	2.0067
		800	1.0101	2.0015	1.0105	2.0107
		1000	1.0120	2.0019	1.0110	2.0110
1	2.5	50	0.9909	2.5150	1.0097	2.5095
		100	0.9935	2.5087	1.0061	2.5041
		150	1.0008	2.5012	1.0014	2.5014
		200	1.0012	2.5071	1.0012	2.5073
		300	1.0062	2.5092	1.0032	2.5083
		500	1.0102	2.5107	1.0082	2.5103
		800	1.0112	2.5182	1.0012	2.5109
		1000	1.0156	2.5208	1.0094	2.5119
1	3	50	0.9909	3.0050	1.0097	3.0010
		100	0.9935	3.0070	1.0061	3.0070
		150	1.0008	3.0058	1.0014	3.0090
		200	1.0006	3.0100	1.0012	3.0100
		300	1.0011	3.0078	1.0012	3.0107
		500	1.0032	3.0087	1.0012	3.0078
		800	1.0007	3.0067	1.0012	3.0076

The simulation study results, as presented in Table 1, demonstrate that the estimates obtained through Bayesian methods closely align with those from the MLE and show good agreement when the sample size is small. In the case of small sample sizes, the Bayesian estimators were found to provide a better fit than the MLE.

However, as the sample size increases, the MLE consistently outperformed the Bayesian estimators in terms of better fitting. The error accumulation, represented by the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE), both decrease as the sample size increases, indicating an improvement in estimation accuracy for both methods. With small sample sizes, the Bayesian approach demonstrates competitive performance and shows promise in accurately estimating the parameters of the Weibull distribution. Nevertheless, as the sample size grows, the MLE tends to yield superior results in terms of fitting the data. The reduction in error accumulation with increasing sample size indicates the benefits of having larger datasets for more precise parameter estimation.

These findings provide valuable insights into the behavior of Bayesian and MLE estimators under different sample size scenarios, particularly when dealing with financial and investment datasets. Understanding the strengths and limitations of each approach is crucial in choosing the most appropriate parameter estimation method for specific applications in finance and insurance domains.

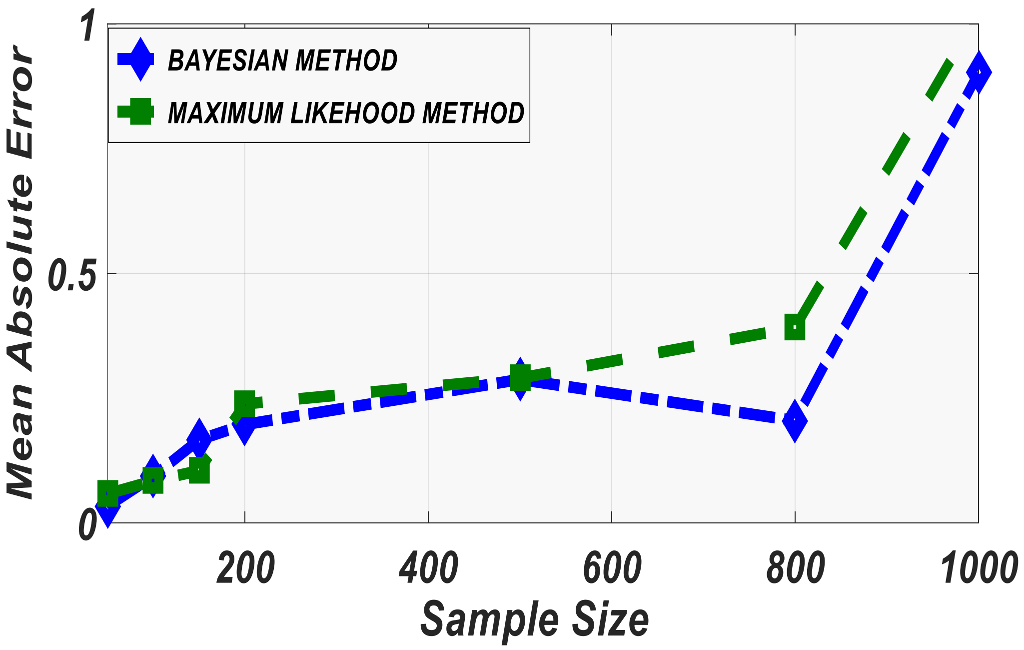

Figure 1. MAE of Bayesian Maximum likelihood Methods for $\alpha = 1$ and $\beta = 1$

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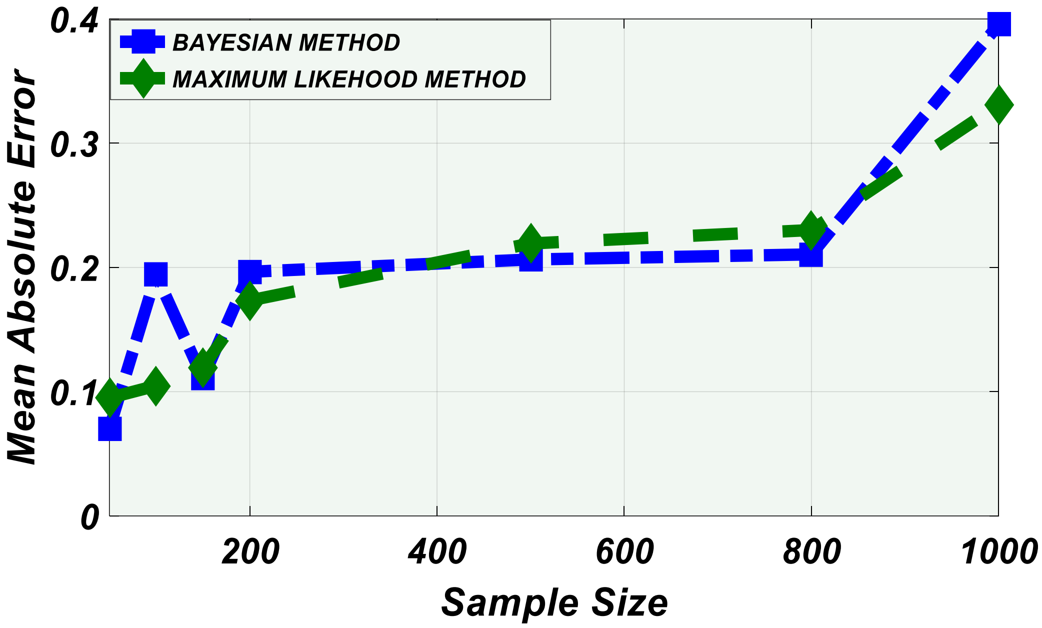

Figure 2. MAE of Bayesian Maximum likelihood Methods for $\alpha = 1.5$ and $\beta = 1$

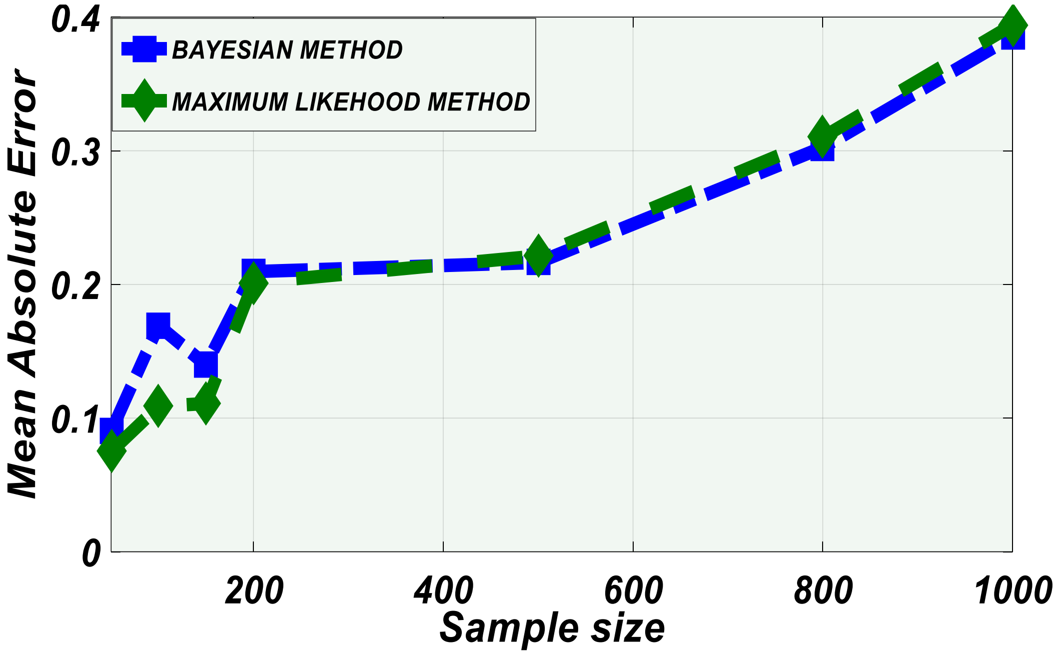

Figure 3. MAE of Bayesian Maximum likelihood Methods for $\alpha = 2$ and $\beta = 1$

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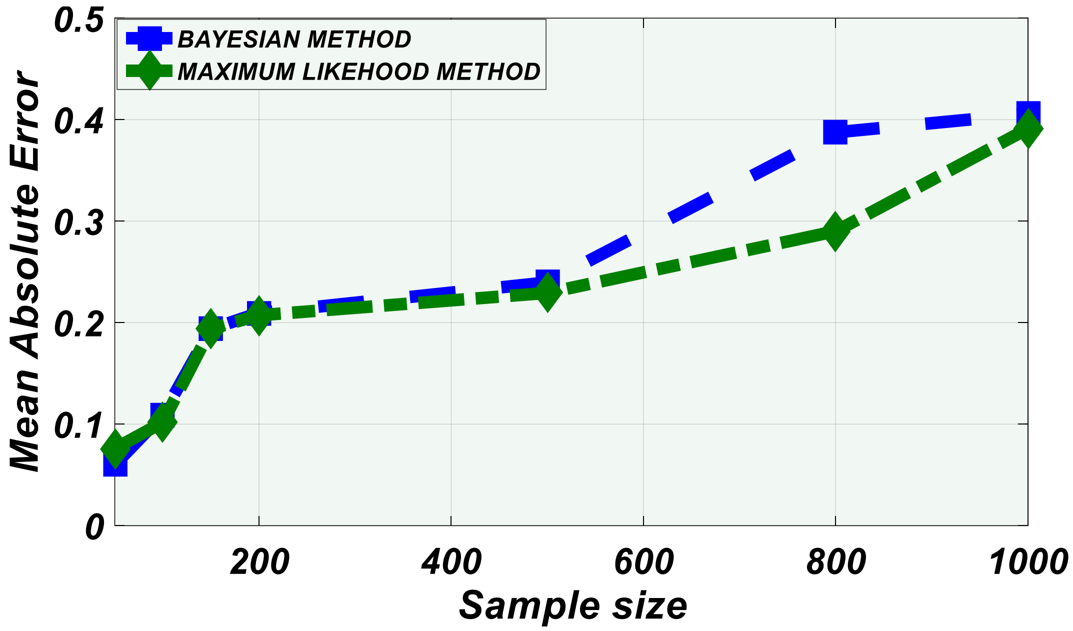

Figure 4. MAE of Bayesian Maximum likelihood Methods for $\alpha = 2.5$ and $\beta = 1$

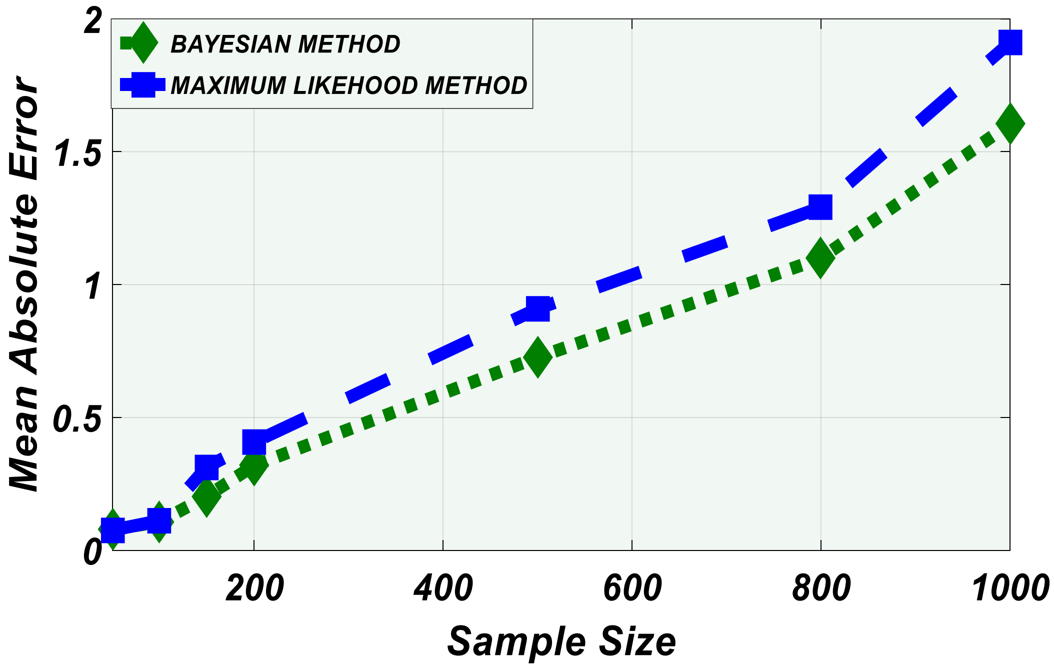

Figure 5. MAE of Bayesian Maximum likelihood Methods for $\alpha = 3$ and $\beta = 1$

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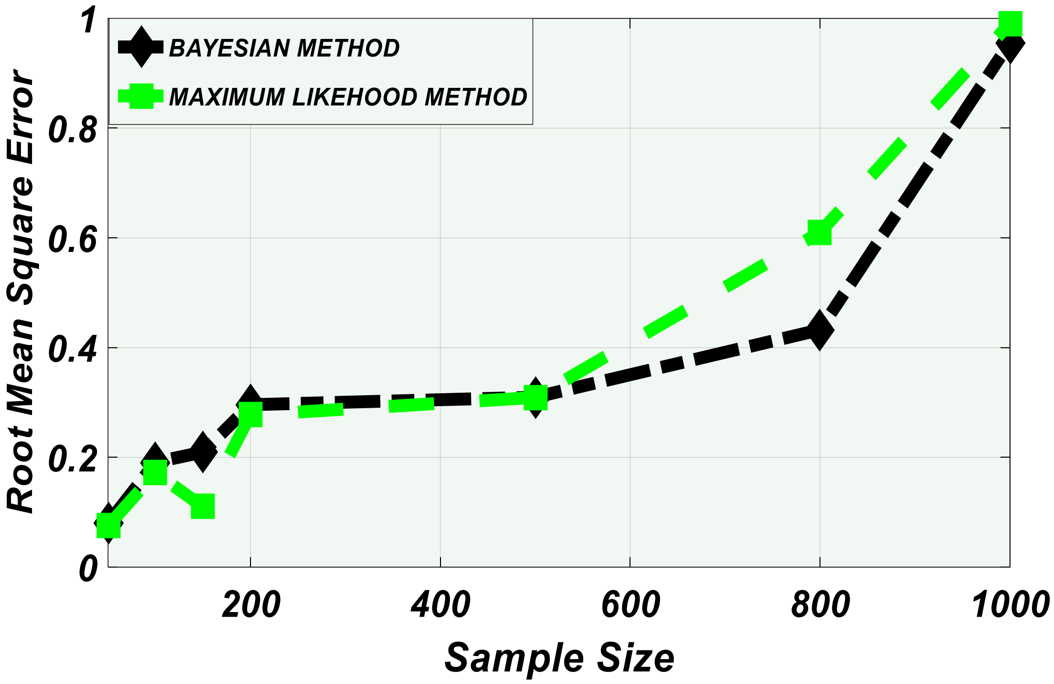

Figure 6. RMSE of Bayesian and Maximum likelihood Methods for $\alpha = 1$ and $\beta = 1$

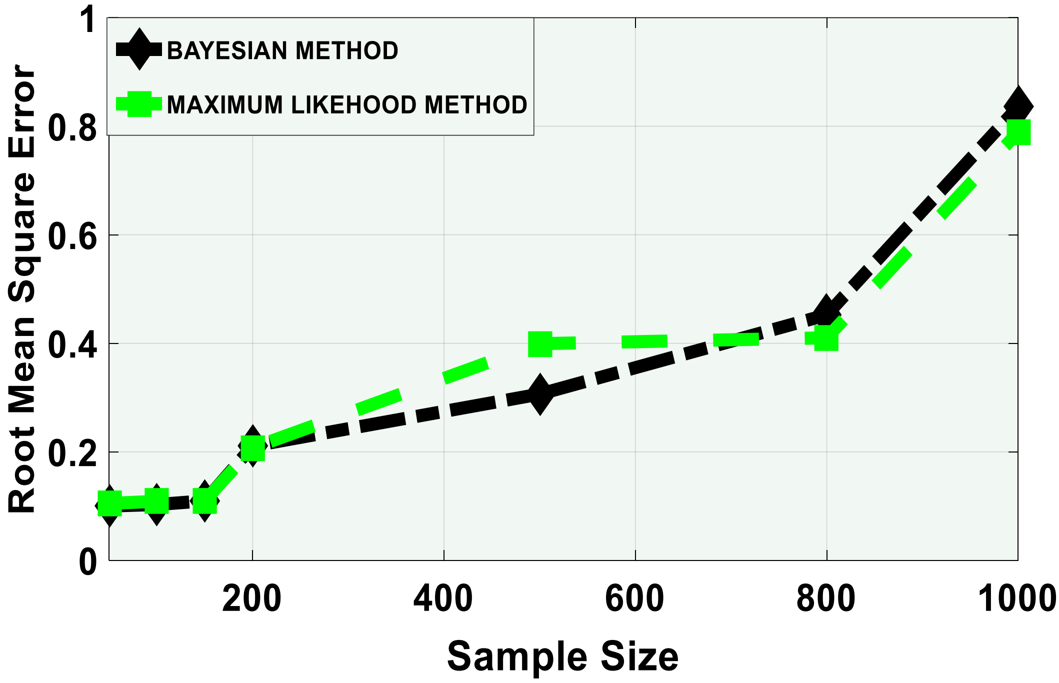

Figure 7. RMSE of Bayesian and Maximum likelihood Methods $\alpha = 1.5$ and $\beta = 1$

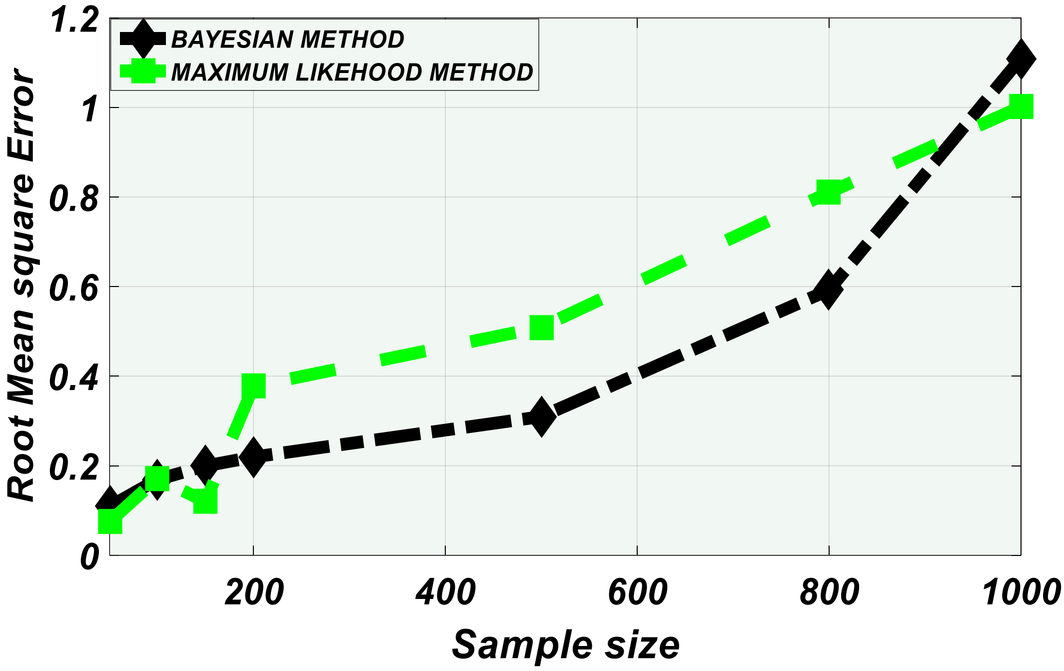

Figure 8. RMSE of Bayesian and Maximum likelihood Methods $\alpha = 2$ and $\beta = 1$

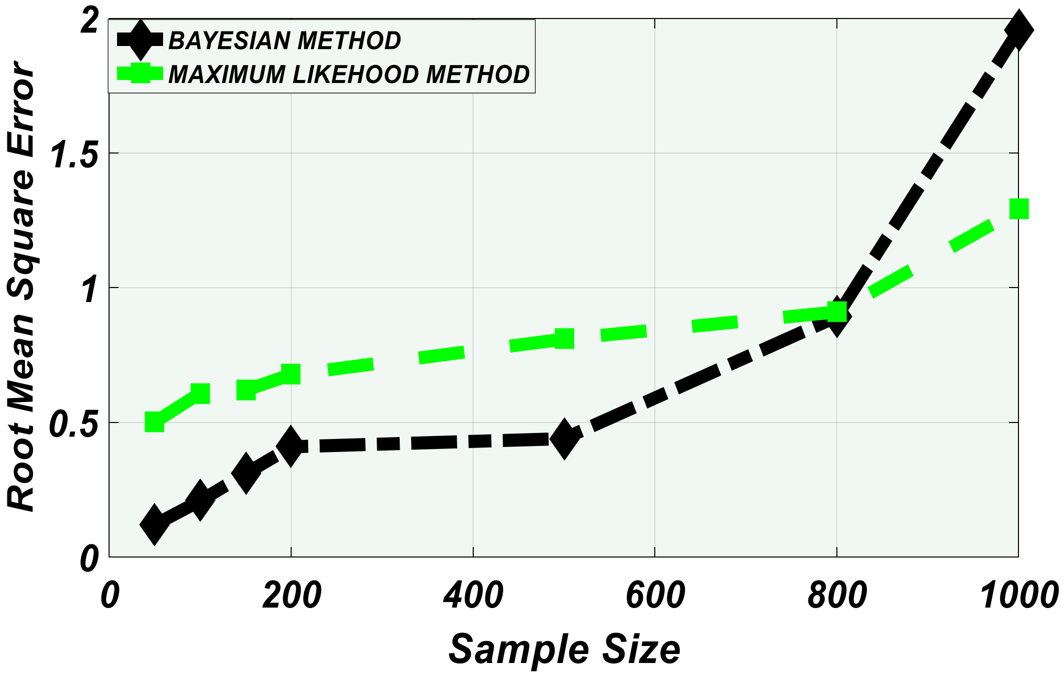

Figure 9. RMSE of Bayesian and Maximum likelihood Methods $\alpha = 2.5$ and $\beta = 1$

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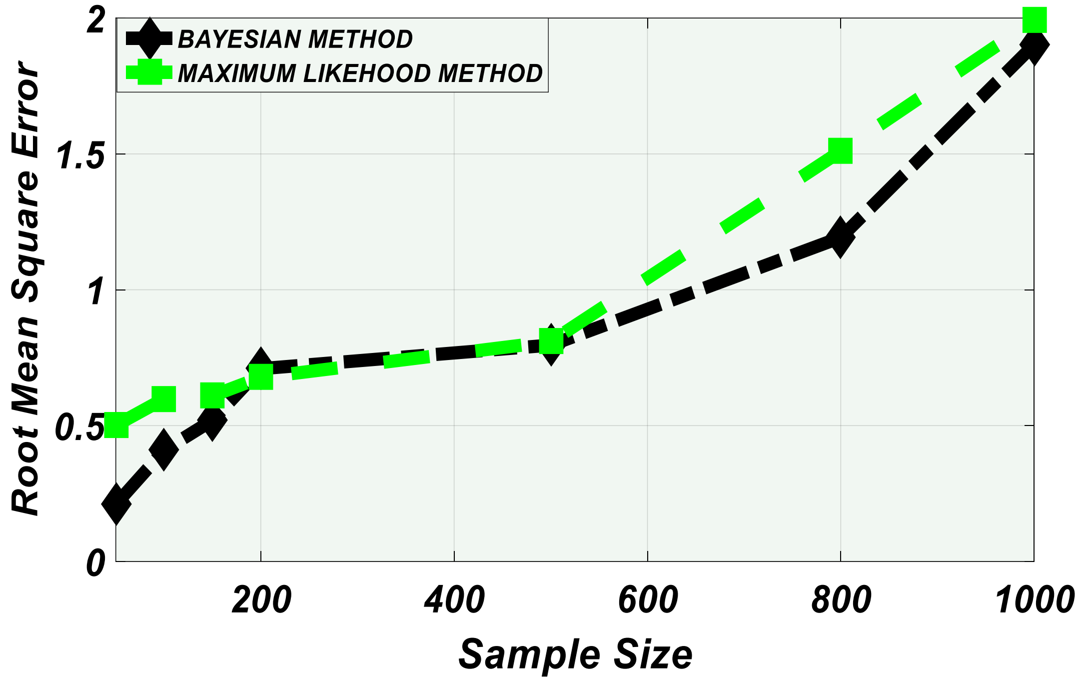

Figure 10. RMSE of Bayesian and Maximum likelihood Methods $\alpha = 3$ and $\beta = 1$

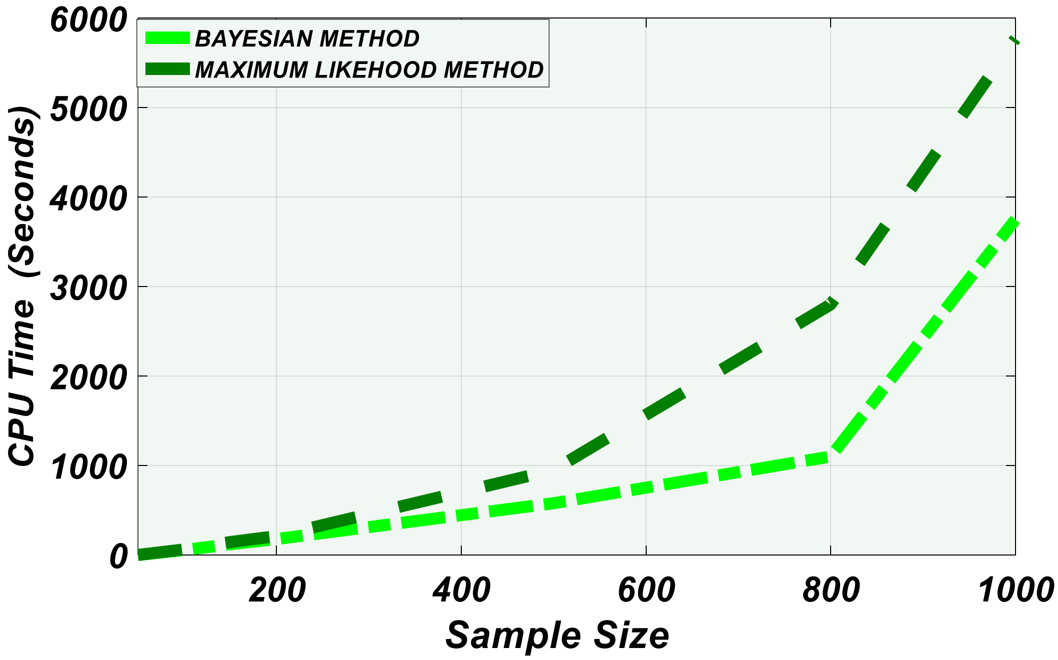

Figure 11. CPU time of Bayesian and Maximum likelihood Methods

Figures 1 to 11 provide a comprehensive analysis of error accumulation and computational time for Bayesian and maximum likelihood estimates across varying sample sizes, using simulated financial data. The results shed light on the behavior of error accumulation in both estimation methods and their suitability for modeling financial data.

For small sample sizes, Figures 1 to 11 demonstrate that the RMSE and MAE values for Bayesian and maximum likelihood estimates exhibit similar patterns of error accumulation. This indicates that both methods perform comparably in capturing the underlying parameters of the Weibull distribution when data availability is limited.

As the sample size increases, a notable reduction in error accumulation is observed for both Bayesian and maximum likelihood approaches. This improvement in estimation accuracy with larger sample sizes signifies the benefit of having more data points to inform parameter estimation.

Additionally, Figures 1 to 11 reveal that for small sample sizes, both Bayesian and maximum likelihood estimators provide a better fit compared to Bayesian Estimator. This suggests that Bayesian methods can offer reliable estimates even with limited data, aligning well with the accuracy of the maximum likelihood method.

In conclusion, the simulation study highlights the strengths of both Bayesian and maximum likelihood estimation approaches in handling financial data. While Bayesian methods demonstrate competitive performance with small sample sizes, the accuracy of both methods improves significantly as the sample size grows. These findings underscore the importance of considering the sample size and data availability when choosing the most appropriate estimation method for financial modeling tasks.

CONCLUSION

Statistical distributions are fundamental in financial sciences for data modeling and analysis. This article utilizes the Weibull distribution, assuming the scale parameter follows a Gamma distribution, to model insurance claim sizes. Parameters were estimated using the Bayesian approach and the Maximum Likelihood estimator, compared through a simulation study across various sample sizes and parameter values. Weibull distributions prove valuable in analyzing insurance claims, providing a concise representation for claims analysis and reserve calculations without extensive raw claims data. Both Bayesian and maximum likelihood methods yield reliable estimates, particularly for lower claims probabilities, benefiting risk management in insurance. Organizations can choose estimation methods based on their specific needs, such as focusing on low claims probabilities. Future research can explore the Weibull distribution with specific parameter distributions. Overall, these estimation methods contribute effectively to decision-making and risk management in the insurance industry.

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